Notes on counting and L-analyticity

Workshop *The meaning of numerals*
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A novel observation

The numeral **zero** cannot be modified by the superlative quantifier **at least** if it refers to the scalar endpoint 0:

(1) a. there are at least two students in the classroom
b. *there are at least zero students in the classroom

To support the empirical claim in (1), we conducted an experiment on Amazon MTurk. 32 English speakers rated the naturalness of 4 sentences like (1a) and (1b) on a 4-point scale. Sentences with **at least two** received the highest score 4 ('natural') by $\geq$ 50% of all subjects, while sentences with **at least zero** received the two lowest scores 2 and 1 ('weird') by $\geq$ 50% of all subjects. The difference in the means of the scores (3.4 v 2.0) is highly significant ($p < 2.2^{-16}$). See Table 1.

Figure 1: Boxplot of **at least 2** and **at least 0**

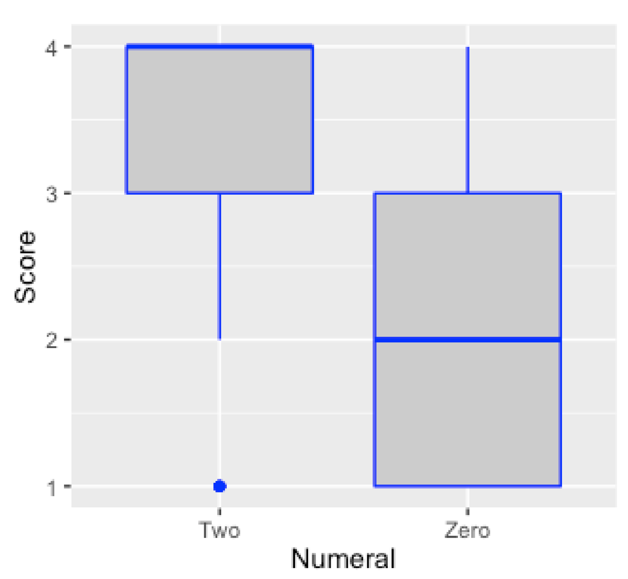
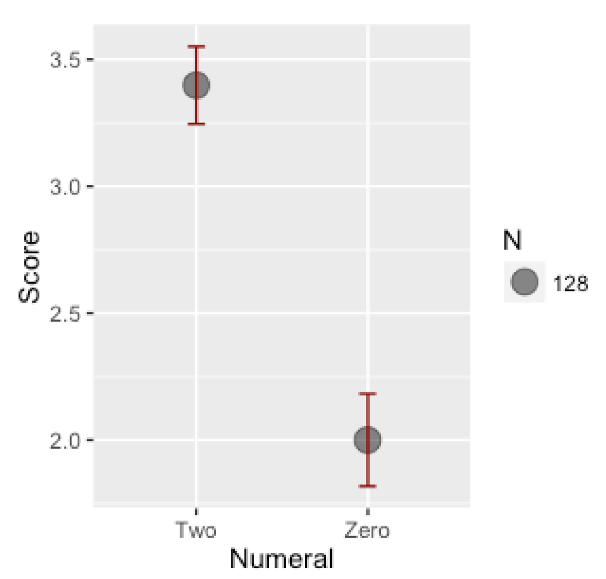


Figure 2: Means of **at least 2** and **at least 0**



L-analyticity

Deviance may result from the sentence being "L-analytical," i.e. tautological or contradictory purely by virtue of the configuration of logical constants contained in it (Barwise and Cooper, 1981; von Fintel, 1993; Gajewski, 2003; Chierchia, 2006; Abrusán, 2007; Gajewski, 2009; Abrusán, 2011)

(2) a. there is a student $\Leftrightarrow \exists x(x \in S \wedge x \in E)$
b. *there is every student $\Leftrightarrow \forall x(x \in S \rightarrow x \in E) \Leftrightarrow_L \top$

(3) a. everyone but Bill danced
$\Leftrightarrow \forall x(x \notin \{b\} \rightarrow x \in D) \wedge \forall P(\forall x(x \notin P \rightarrow x \in D) \rightarrow \{b\} \subseteq P)$
$\Rightarrow \forall x(x \notin \{b\} \rightarrow x \in D) \wedge \neg\forall x(x \notin \emptyset \rightarrow x \in D)$
b. *someone but Bill danced
$\Leftrightarrow \exists x(x \notin \{b\} \wedge x \in D) \wedge \forall P(\exists x(x \notin P \wedge x \in D) \rightarrow \{b\} \subseteq P)$
$\Rightarrow \exists x(x \notin \{b\} \wedge x \in D) \wedge \neg\exists x(x \notin \emptyset \wedge x \in D)$
$\Leftrightarrow_L \bot$

The L-analyticity of "at least zero"

To derive the deviance of (1b), we assume that **at least *n*** denotes the GQ in (4), where ***n*** is a bare numeral and n the scale point that it refers to.

(4) $[\![$**at least *n***$]\!](P)(Q) = 1$ iff $|P \cap Q| \geq n$ (assumption for exposition simplicity)

Given (4), (1b) is analytically true and, moreover, L-analytically true, since **at least zero** belongs to the "logical skeleton" of (1b) (Fox and Hackl, 2006)) and denotes a constant function with value 1 (truth):

(5) *there are at least zero students in the classroom
$\Leftrightarrow |S \cap C| \geq 0$
$\Leftrightarrow_L \bot$

Thus, the deviance of (1b) is due to it being an L-analytical sentence.

Implications for the theory of exhaustification and L-analyticity

We show that exhaustification cannot obviate ungrammaticality induced by L-analyticity. First, we observe that the structure in (6a), which contains **exh**$_C$ (Fox, 2007), has the same truth condition as the corresponding structure without **exh**$_C$, see (6c), given that the domain C is the set in (6b) (or any other set containing only alternatives that contradict each other). Thus, (1b) does not provide a testing ground for our exploration.

(6) *there are at least zero students (in the classroom)
a. $[_\psi$ exh$_C$ $[_\phi$ there are at least zero students$]]$
b. C = {there are more than zero students, there are exactly zero students} (non-excludable, non-excludable)
c. $\psi \Leftrightarrow \phi \Leftrightarrow_L \top$

However, we observe that deviance persists with embedding of **at least zero** under a universal quantifier, see (7a) and (7b).

(7) a. *every human has at least zero children
b. *you are required to read at least zero books

If exhaustification could obviate ungrammaticality (7a) and (7b) would be expected to be non-deviant. Here is why: given that (7b) has the parse in (8a), it has the non-tautological truth condition in (9a), since the alternatives in C do not contradict each other and are hence innocently excludable, see (9a).

(8) a. $[_\psi$ exh$_C$ $[_\phi$ you are required to read at least zero books$]]$
b. C = {$\Box$ you read more than zero books, $\Box$ you read exactly zero books} (excludable, excludable)
c. $\psi \Leftrightarrow \Diamond$ you read exactly 0 books $\wedge \Diamond$ you read more than 0 books $\not\Leftrightarrow \top$

$\Rightarrow$ L-analyticity cannot be obviated by embedding under **exh**.

Further support for our conclusion: the L-analytically true expression ***there is every student** is not salvaged by exhaustification:

(9) a. $[_\psi$ exh$_C$ $[_\phi$ there is every student$]]$ b. C = {there is a student} (excludable)
c. $\psi \Leftrightarrow \forall x(x \in S \rightarrow x \in E) \wedge \neg\exists x(x \in S \wedge x \in E) \not\Leftrightarrow \top$

$\Rightarrow$ L-analyticity is a type of ungrammaticality: it behaves like other types of ungrammaticality, which are also not salvageable by syntactic embedding.

A prediction

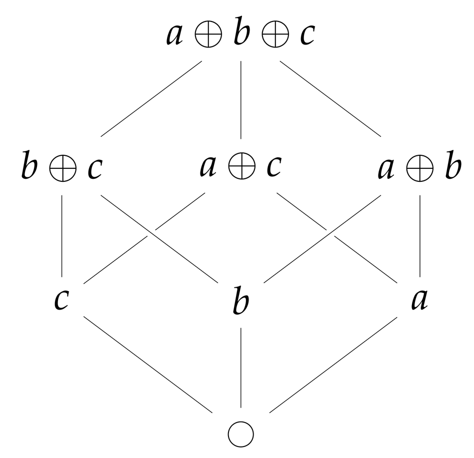
We predict that the *meaning* of (7a) can be felicitously expressed by a non-L-analytical sentence, such as (10a), whose LF is (10b) (Hurford, 1974; Chierchia et al., 2012; Fox and Spector, 2018).

(10) a. every human has zero or more children
b. exh$_C$ [every human]$_x$ [[x has zero children] or [x has more than zero children]]

To test this prediction, we conducted an experiment on Amazon MTurk. Specifically, we tested the claim that **every...at least zero** has a different status from **every...zero or more**, **every...at least two**, and **every...two or more** (157 subjects giving 1 'weird'/'not weird' judgment per sentence type). The proportion of 'weird' responses to **every...at least zero** is greater than that to its **every...zero or more** counterpart (40% and 28%, respectively, $p = 0.01605$). In contrast, the proportions of 'weird' responses to **every...at least two** and **every...two or more** are equal (7% and 12%, respectively, $p = 0.34$) and smaller from **every...at least zero** and **every...zero or more**.

Implications for the semantics of the bare numeral zero

Bylinina & Nouwen (2018) (henceforth *B&N*) argue that *every* plural noun has in its the denotation a special element, $\bigcirc$, whose atoms count 0.



$[\![$two students$]\!] = [\lambda x.\ x \in [\![$student$]\!] \wedge \#x = 2]$
$= \{a \oplus b, b \oplus c, a \oplus c\}$

$[\![$zero students$]\!] = [\lambda x.\ x \in [\![$student$]\!] \wedge \#x = 0] = \{\bigcirc\}$

(11) there are n students $\Leftrightarrow \exists x(x \in [\![$students$]\!] \wedge \#x = n)$

Moreover, B&N argue that parsing (12) with exh$_C$, as in (12b) rescues it from being an L-analytical sentence, assuming **zero** alternates with other numerals.

(12) there are zero students
a. $[_S$ there are zero students$]$
$\Leftrightarrow \exists x(\#x = 0 \wedge x \in [\![$students$]\!]) \Leftrightarrow_L \top$
b. $[_S$ exh$_C$ $[_S$ there are zero students$]]$
$\Leftrightarrow \exists x(\#x = 0 \wedge x \in [\![$students$]\!]) \wedge \neg\exists x(\#x > 0 \wedge x \in [\![$students$]\!]) \not\Leftrightarrow \top$

However, the data discussed before suggest that L-analyticity cannot be obviated by exhaustification. Therefore, we reject B&N's assumption that the maximality/exhaustive aspect of the meaning of the bare numeral **zero** is syntactically represented as in (12b).

Are there better theories of "zero"?

Suppose numerals have a two-sided meaning as a matter of semantic content (Breheny, 2008; Geurts, 2006; Kennedy, 2015). We correctly derive that **there are zero students** is non-tautological, and that **there are at least zero students** is L-tautological.

(13) a. there are 0 students $\Leftrightarrow$ exh$_C$(there are zero students)
$\Leftrightarrow \max\{n \mid \exists x[x \in [\![$students$]\!] \wedge \#x = n]\} = 0 \not\Leftrightarrow \top$
b. there are at least 0 students $\Leftrightarrow$ exh$_C$(there are at least zero students)
$\Leftrightarrow \max\{n \mid \exists x[x \in [\![$students$]\!] \wedge \#x = n]\} \geq 0 \Leftrightarrow_L \top$

However, we still derive, incorrectly, that the deviance of **at least zero** is obviated under universals:

(14) exh$_C$ ($\Box$(there are at least zero students))
$\Leftrightarrow \Box(\max\{n \mid \exists y[y \in [\![$students$]\!] \wedge \#y = n]\} \geq 0])$
$\wedge\neg\Box(\max\{n \mid \exists y[y \in [\![$students$]\!] \wedge \#y = n]\} = 0])$
$\wedge\neg\Box(\max\{n \mid \exists y[y \in [\![$students$]\!] \wedge \#y = n]\} > 0])$
$\not\Leftrightarrow \top$

The logical status of scales and approximators

The scale that the semantic evaluation of **at least zero** is based on matters for the (un)grammaticality of expressions in which **at least zero** occurs. That is, if 0 is not a scalar endpoint, as is the case for the Celsius scale in (15b), no deviance arises.

(15) a. *There are at least zero students in the classroom
b. The temperature is at least zero degrees Celsius
c. *The temperature is at least zero degrees Kelvin

The contrast between (15a) and (15b) mirrors the contrast between ***approximately zero students** and **approximately zero degrees celsius** (Solt, 2014):

(16) a. *There are approximately zero students in the classroom
b. The temperature is approximately zero degrees Celsius

We leave for future research to determine if the triviality of applying an approximator to **zero students** can be conceived of as L-analyticity.