At least vs. *more than*: Explaining ignorance in terms of relevance of speaker belief

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The meaning of numerals
ZAS, Berlin
March 27, 2019

Empirical background: ignorance inferences (IIs)

Nouwen (2010), Nouwen (2015): *at least* obligatorily conveys (speaker) ignorance, while *more than* doesn't.

(1) a. A hexagon has more than 2 sides.
b. #A hexagon has at least 2 sides.

(2) a. I own more than 2 dogs.
b. #I own at least 2 dogs.

We aim to derive the contrast between the (a) and (b) sentences from just the difference in truth-conditional content ('$>$' vs. '$\geq$') between *more than* and *at least*.

If we succeed, we derive Nouwen (2010)'s class A/class B distinction: the dividing line between the two classes follows the strict/non-strict comparison distinction.

The source of IIs – *or*

Sauerland (2004): IIs arise from symmetric alternatives.

(3) #I own a bird or a cat. $b \vee c$

- Alt($b \vee c$) includes both b and c.
- $(b \vee c) \wedge \begin{cases} \neg b & \Rightarrow & c \\ \neg c & \Rightarrow & b \end{cases}$ "Symmetry"

  Hence, $(b \vee c) \wedge \neg b \wedge \neg c \equiv \bot$.
- Therefore, no scalar inferences (SIs) from these alternatives, but epistemic inferences: $\neg Kb$ and $\neg Kc$
- $K(b \vee c) \wedge \begin{cases} \neg Kb & \Rightarrow & \neg K \neg c \\ \neg Kc & \Rightarrow & \neg K \neg b \end{cases}$ "Epistemic symmetry"
- Overall conveyed meaning: $K(b \vee c) \wedge Ib \wedge Ic \wedge K\neg(b \wedge c)$ ✓

  $(I\phi \equiv \neg K\phi \wedge \neg K\neg\phi)$

The source of IIs – *at least*

Büring (2008): The same is true for *at least*.

(2b) I own at least 2 dogs. $[\geq 2]$

- Alt($[\geq 2]$) includes both $[>2]$ and $[=2]$.
- $[\geq 2] \wedge \begin{cases} \neg[>2] & \Rightarrow & [=2] \\ \neg[=2] & \Rightarrow & [>2] \end{cases}$ Symmetry

  Hence, $[\geq 2] \wedge \neg[>2] \wedge \neg[=2] \equiv \bot$.
- Therefore, no scalar inferences (SIs) from these alternatives, but epistemic inferences: $\neg K[>2]$ and $\neg K[=2]$
- $K[\geq 2] \wedge \begin{cases} \neg[K>2] & \Rightarrow & \neg K\neg[=2] \\ \neg[K=2] & \Rightarrow & \neg K\neg[>2] \end{cases}$ Epistemic symmetry
- Overall conveyed meaning: $K[\geq 2] \wedge I[>2] \wedge I[=2]$ ✓

The problem raised by *more than*

Problem: Along the same lines, we also derive IIs for more than (cf. Mayr, 2013).

(2a) I own more than 2 dogs. $[>2]$

- Alt($[>2]$) includes both $[>3]$ and $[=3]$.
- The counting scale is non-dense.
- $[>2] \wedge \begin{cases} \neg[>3] & \Rightarrow & [=3] \\ \neg[=3] & \Rightarrow & [>3] \end{cases}$ Symmetry

  Hence, $[>2] \wedge \neg[>3] \wedge \neg[=3] \equiv \bot$.
- Therefore, no scalar inferences (SIs) for these alternatives, but epistemic inferences: $\neg K[>3]$ and $\neg K[=3]$
- $K[>2] \wedge \begin{cases} \neg[K>3] & \Rightarrow & \neg K\neg[=3] \\ \neg[K=3] & \Rightarrow & \neg K\neg[>3] \end{cases}$ Epistemic symmetry
- Overall conveyed meaning: $K[>2] \wedge I[>3] \wedge I[=3]$ ✗

Non-solution: Fewer alternatives

(2a) I own more than 2 dogs. $[>2]$

- Simple idea: $\mathrm{Alt}([>2])$ includes $[>3]$ but excludes $[=3]$.
- No symmetry, hence no epistemic inferences that lead to IIs
- BUT: No symmetry, hence a SI for (2a): $[>2] \wedge \neg[>3]$
- Overall conveyed meaning: $K[=3]$ ✗

Solution: Even fewer alternatives

(2a) I own more than 2 dogs. $[>2]$

- Another simple idea: $\text{Alt}([>2]) = \{\}$ (Cremers et al., 2017).
- No alternatives, hence no SIs
- No alternatives, hence no epistemic inferences
- Overall conveyed meaning: $K[>2]$ ✓
- BUT: the initial observation about *at least* and *more than* is just restated in terms of stipulations about alternatives:

  $\text{Alt}([\geq 2]) = \{[>2], [=2]\}$

  $\text{Alt}([>2]) = \{\}$ (✗)
- Can we derive these sets in a principled way?

Our solution (ingredient 1): More (formal) alternatives

- We adopt the UDM (Fox&Hackl 2006): all scales are dense.
- Immediate consequence of the UDM:

  $K[>2] \wedge \left\{ \begin{array}{lcl} \neg K[>3] & \not\Rightarrow & \neg K \neg [=3] \\ \neg K[=3] & \not\Rightarrow & \neg K \neg [>3] \end{array} \right.$ No epistemic symmetry (✓)

  (This is crucial but not enough on its own, as we will see.)
- Specifically, we assume:

  $\text{F-Alt}([>2]) = \{[\geq n] : n \in \mathbb{Q}^+\} \cup \{[>n] : n \in \mathbb{Q}^+\} \cup \{[=n] : n \in \mathbb{Q}^+\}$

  (Alt = F-Alt ∩ R_c, 'F' for formal; R_c = the set of all relevant propositions in context c)
- The result of Fox&Hackl (2006) persists for our more inclusive set F-Alt:

  $[>2] \wedge \neg \bigvee \{\phi \in \text{F-Alt} : [>2] \wedge \neg\phi \not\equiv \bot\} \equiv \bot$

  Thus, exhaustification cannot apply, hence no SI. ✓
- Moreover, we assume uniformity: $\text{F-Alt}([\geq 2]) = \text{F-Alt}([>2])$

A problem and its solution (our ingredient 2)

- With the UDM, we derive many unwanted uncertainty inferences if all formal alternatives are in R_c:
  - In the the standard grammatical approach (Chierchia et al. 2012; Fox 2007), we derive for both $[> 2]$ and $[\geq 2]$ ignorance about all stronger alternatives (total ignorance). (✗)
  - In a Neo-Gricean approach (e.g. Schwarz, 2016), we derive for $[> 2]$ uncertainty about all stronger alternatives. (✗)
- Can we derive that for all c, none of the stronger alternatives is in R_c?
- In the grammatical approach, we can: by closing relevance under speaker belief. ✓

Standard grammatical theory: Main ingredients

- The "basic" Gricean maxims (Quality and Quantity)
- Closure conditions of relevance
- Grammatical exhaustification operator, *exh*, that quantifies over formal alternatives

Gricean maxims

(3) **Maxim of Quality**
The speaker should only utter sentences that the speaker believes to be true. ("Tell nothing but the truth.")

(4) **Maxim of Quantity**
The speaker should utter a sentence S such that, for every relevant proposition ϕ that the speaker believes to be true, S entails ϕ. ("Tell the whole truth.")

(In what follows, we only consider contexts in which both maxims are active.)

Closure conditions of relevance

(5) **Closure conditions of relevance**

a. If ϕ is relevant, then so is $\neg\phi$.

b. If ϕ and ψ are both relevant, then so is $\phi \wedge \psi$.

Grammatical exhaustification device: *exh*

- *exh* may attach to any propositional constituent (i.e. any node of type *st*).
- *exh* in [exh S] quantifies over Alt(S) (as characterized before).
- [exh S] is true iff S is true and all innocently excludable (IE) alternatives in Alt(S) are false.
- An alternative S′ in Alt(S) is IE iff $S \wedge \neg S'$ doesn't entail any disjunction of alternatives in $\text{Alt}(S) \setminus \{S\}$.
- Innocent exclusion takes care of symmetric alternatives but cannot deal with dense alternatives (Gajewski, 2009):

$$\text{exh}(\{b, c, \ldots\})(b \vee c) \not\equiv \bot \quad ✓$$

$$\text{exh}(\{[> n] : n \in \mathbb{Q}^+\} \cup \ldots)([> 2]) \equiv \bot \quad ✓$$

Generalization 1

(6) **Definition of settle**
S settles ϕ just in case $[\![S]\!] \vDash \phi$ or $[\![S]\!] \vDash \neg\phi$.

(7) **Generalization 1**
For any context c, uttered sentence S, and proposition ϕ, if ϕ is relevant in c and S doesn't settle ϕ, then S gives rise to an inference of speaker ignorance about ϕ.

Proof:

- If (i) ϕ is relevant and not settled by S, then (ii) so is $\neg\phi$.
- From (i), Quantity allows to infer $\neg K\phi$, and from (ii), $\neg K\neg\phi$.
- Thus, overall Quantity yields $I\phi$.

Modified numerals: Too many ignorance inferences

(8) exh $[_S$ I own at least 2 dogs $]$

$\text{Alt}(S) = \{[\geq n] : n \in \mathbb{Q}^+\} \cup \{[> n] : n \in \mathbb{Q}^+\} \cup \{[= n] : n \in \mathbb{Q}^+\}$.

- Because of symmetry: $[\![(8)]\!] = [\geq 2]$. ✓
- BUT: $[\geq 2]$ doesn't settle $[> n]$, $[= n]$, for all $n \geq 2$.
- Hence, by quantity reasoning and the closure conditions: ignorance for $[> n]$, $[= n]$ for all $n \geq 2$ (total ignorance instead of partial ignorance, i.e., ignorance about about just $[> 2]$ and $[= 2]$; Schwarz, 2016). ✗

(9) ~~exh~~ $[_S$ I own more than 2 dogs $]$

Alt(S) = the same as above.

- Because of density: $[\![(9)]\!] = [> 2]$. ✓
- BUT: $[> 2]$ doesn't settle $[> n]$, $[= n]$, for all $n > 2$.
- Hence, by quantity reasoning and the closure conditions: ignorance for $[> n]$ and $[= n]$ for all $n > 2$. ✗

The missing ingredient: Relevance of belief

Our minimal extension the standard theory: The speaker's beliefs about the truth of what is relevant are themselves relevant.

(10) **Closure conditions of relevance (revised)**

a. If ϕ is relevant, then so is $\neg\phi$.

b. If ϕ and ψ are both relevant, then so is $\phi \wedge \psi$.

c. If ϕ is relevant, then so is $K\phi$.

Some motivation for closure under belief

Fox (2016): "Silence is uncooperative."

Example: In the context of a murder trial, if the lawyer asks the witness, w, "Where was John at the time of the murder?", w can't just look the lawyer in the eye and remain silent.

Intuition: "If w believes something that bears on John's whereabouts at the time of the murder, w is required to say so. If not, w is required to reveal this lack of opinion."

Generalization 2 (instead of generalization 1)

If ϕ is relevant in c and not settled by S, then either:

1. S entails speaker ignorance about ϕ; or
2. the maxim of quantity licenses the inference of a contradiction (empirically unattested).

Proof:

If ϕ is relevant and not settled, Quantity yields $I\phi$.
$I\phi$ is relevant by the closure conditions.

1. S either entails $I\phi$ (and hence Quantity stays silent on $I\phi$); or
2. Quantity allows to infer $\neg K(I\phi)$.

   This contradicts $K(I\phi)$, which follows from negative introspection (necessary to obey Quality).

Thus, ignorance must be able to be derived in grammar, and it can no longer be derived by quantity reasoning (since that would yield a contradiction).

That is, ignorance must be derived in grammar alone.

A consequence for relevance

(11) **A consequence of closing relevance under belief**
If a sentence S doesn't settle or entail ignorance about ϕ, then ϕ isn't relevant in c.

We'll soon see that certain propositions can't be relevant (to certain sentences) in any context.

We call such propositions obligatorily irrelevant.

Matrix K

Since ignorance must be derived in grammar, language must make available a covert belief operator, e.g. the syntactic item K proposed by Meyer (2013).

(12) **Matrix K**
An (assertive) sentence must be c-commanded by an occurrence of K, where $[\![\mathrm{K}]\!] = K$.

Given this, together with our assumption about *exh*, it follows that *exh* can apply both above and/or below K.

At least - what's settled or entailed ignorance about

(13) exh $[_{S_2}$ K exh $[_{S_1}$ I own at least 2 dogs]]

- $exh(\text{Alt}(S_2))(K[\geq 2]) \Rightarrow K[\geq 2] \wedge \neg K[> 2] \wedge \neg K[= 2]$.

  By the first conjunct, $[\geq 2]$ is settled
  (given speaker competence).
- $K[\geq 2] \wedge \begin{cases} \neg K[> 2] & \Rightarrow & \neg K \neg [= 2] \\ \neg K[= 2] & \Rightarrow & \neg K \neg [> 2] \end{cases}$ Epistemic symmetry
- Hence, $exh(\text{Alt}(S_2))(K[\geq 2]) \Rightarrow K[\geq 2] \wedge I[> 2] \wedge I[= 2]$.
- Thus, $[> 2]$ and $[= 2]$ are entailed ignorance about.
- Therefore, overall $[\geq 2]$, $[> 2]$, and $[= 2]$ can be relevant
  ($+$ the weaker alternatives of $[\geq 2]$).

At least - what's not settled or entailed ignorance about

(13) exh $[_{S_2}$ K exh $[_{S_1}$ I own at least 2 dogs]]

Consider $K[>3]$, $K[=3]$:

- $K[\geq 2] \wedge \begin{cases} \neg K[>3] & \not\Rightarrow & \neg K \neg[=3] \\ \neg K[=3] & \not\Rightarrow & \neg K \neg[>3] \end{cases}$ (because of density)

  No epistemic symmetry
- More generally, no epistemic symmetry for $K[>3]$, $K[=3]$ relative to any subset of F-Alt($[\geq 2]$) (because of density)
- Thus, $[>3]$ and $[=3]$ is neither settled nor entailed ignorance about.
- Therefore, $[>3]$, and $[=3]$ cannot be relevant in any context (and the same for all other $n \geq 2$).
- That is, except for $[>2]$ and $[=2]$ all stronger alternatives turn out to be obligatorily irrelevant.

More than - nothing's entailed ignorance about

(14) exh $[_{S_2}$ K ~~exh~~ $[_{S_1}$ I own more than 2 dogs]]

Consider $K[>3]$, $K[=3]$:

- $K[>2] \wedge \begin{cases} \neg K[>3] & \not\Rightarrow & \neg K\neg[=3] & \text{(because of density)} \\ \neg K[=3] & \not\Rightarrow & \neg K\neg[>3] & \text{(because of density)} \end{cases}$

  No epistemic symmetry
- More generally, no epistemic symmetry for $K[>3]$, $K[=3]$ relative to any subset of F-Alt($[>2]$) (because of density)
- Thus, $[>3]$ and $[=3]$ is neither settled nor entailed ignorance about.
- Therefore, $[>3]$, and $[=3]$ cannot be relevant in any context (and the same for all other $n \geq 2$).
- That is, without any exception all stronger alternatives turn out to be obligatoriy irrelevant (because, other than with $[\geq 2]$, epistemic symmetry can never arise).

Empirical predictions

(1) a. A hexagon has more than 2 sides.
$K[> 2]$ ✓

b. # A hexagon has at least 2 sides.
$K[\geq 2] \wedge I[= 2] \wedge I[> 2]$ ✓

(2) a. I own more than 2 dogs.
$K[> 2]$ ✓

b. # I own at least 2 dogs.
$K[\geq 2] \wedge I[= 2] \wedge I[> 2]$ ✓

Caveat: To derive that *at least n* obligatorily conveys ignorance, we need to stipulate that $[= n]$ and $[> n]$ are always relevant.

Empirical predictions, cont'd

(15) Context: Ann's diet requires that she eat no more than two cookies.

Q. Did Ann follow her diet?
$\{[\leq 2], [> 2]\}$

A_1. No, she ate at least 3 cookies.
$K[\geq 3] \wedge I[= 3] \wedge I[> 3]$ ✓

A_2. No, she ate more than 2 cookies.
$K[> 2]$ ✓

Importantly, both $K[\geq 3]$ and $K[> 2]$ settle $[\leq 2]$ and $[> 2]$, i.e., they settle Q.

Empirical predictions, cont'd

(16) Context: To win a round, 6 or more clubs are required.

Q. Will you win this round?
$\{[<6],[\geq 6]\}$

A_1. ?Yes, more than 5 of my eight cards are clubs
$K[>5]$ ✓

A_2. Yes, more than 6 of my eight cards are clubs
$K[>6]$ ✓

A_1 is degraded (Cremers et al. 2017).

Obligatory irrelevance explains why: $K[>5]$ neither settles nor entails ignorance about $[<6]$, $[\geq 6]$.

Thus, A_1 renders the question irrelevant (and A_2 doesn't).

Empirical discussion

(17) A. How many dogs does Ann own?

B. Ann owns more than two dogs.
$K[>2]$

Quantitative data suggest that subjects draw from B's response the inference that B is ignorant (Westera & Brasoveanu 2014, Cremers et al. 2017).

How can we account for this?

- (17-B) renders $[> n]$, $[= n]$, $[\geq n]$ for all $n > 2$ irrelevant.
- Thus, B dodges A's question.
- One reason for dodging a question is being ignorant.

Empirical discussion, cont'd

We think that the experiments show that the context of a *how many* question a non-grammatically conveyed inference of ignorance can be made.

The speaker's discourse behavior (dodging the question) suggests ignorance.

We think that such behavior can often interpreted differently from being based in ignorance.

(18) Q. How old are you?

A_1. I'm more than 40.

A_2. #I'm at least 40.

(19) Q. How much did you pay for your new watch?

A_1. I payed more than 1000 Euro.

A_2. #I payed at least 1000 Euro.

Empirical discussion, cont'd

Inferences of ignorance are not tied to linguistic behavior.

Situation: You know that Ann wants to tighten a single bolt on her bicycle. You see that she goes into the shed and comes out with three wrenches of different sizes. What do you conclude from her behavior? (She doesn't know the exact size of the bolt.)

We think that ignorance inferences in the context of *how many* questions might be of that type.

We hope to address this issue in future research.

Discussion

- We suggest to amend the grammatical approach with the UDM and closure of relevance under belief.
- End result: no ignorance (or even $\neg K\phi$) inferences for *more than* and only partial ignorance for *at least*. ✓
- Importantly, the difference between *more than* and *at least* follows from just the difference in their truth conditional content ('$>$' vs. '$\geq$').
- Therefore, our account generalizes to explain the class A/class B distinction.

THANK YOU!

Appendix: Total ignorance and total uncertainty

Evidence against total ignorance inferences being conveyed by at least (Schwarz, 2016):

(20) a. # At least two members of the quintet were born in Germany. Exactly three were born in Canada.

b. At least one member of the quintet was born in Germany. Exactly three were born in Canada.

Evidence against total uncertainty inferences being conveyed by more than:

(21) a. # At least two members of the quintet were born in Germany. To be more precise, exactly three were born in Berlin.

b. More than one member of the quintet was born in Germany. To be more precise, exactly three were born in Berlin.