Transition Tables, Musical Spaces, and Empirical Relatedness Measures between Chords

Assaf Brown
assafbrown@gmail.com
ELSC, Hebrew University; The Department of Cognitive Science, Hebrew University

VIEW NOW

Abstract

I will briefly discuss associations between three objects:

1. Transition tables – describe the frequency of the transition between each pair of chords.
2. Musical spaces – abstract entities that serve music theorists to capture certain relations between chords.
3. Empirical relatedness – judgments of relatedness between each pair of diatonic chords within a given tonal context. Measured by Bharucha and Krumhansl.

I measured the correlations (both Spearman and Pearson correlations) between these three representations and showed that they are (usually) significantly correlated.
Further analysis of the results indicates (among others) that (1) different normalizations may significantly affect the correlations; that (2) 'crossfading' classical and rock corpora causes stronger correlations, suggesting that the feeling of relatedness is affected by both corpora; and that (3) the directionality of the chords definitely matters for the classical corpora but barely for the rock corpus.

Three objects

(1) Transition Table (TT) -

Frequency of transition between every pair of chords in a corpus of Mozart's pieces.

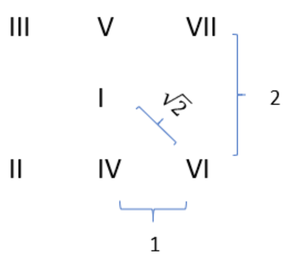
antecedent \ consequent	I	II	III	IV	V	VI	VII
I		.084	.014	.181	.22	.07	.014
II	.083		.005	.004	.078	.005	.006
III	.002	.004		.013	.002	.014	.000
IV	.026	.011	.002		.032	.002	.001
V	.105	.002	.002	.005		.015	.000
VI	.002	.003	.001	.002	.004		.001
VII	.001	.000	.000	.000	.000	.000	

(2) Empirical Relatedness Table (ERT)

Empirical judgments of relatedness between two chords, taken from Barucha and Krumhansel (1983)

first chord \ Second chord	I	II	III	IV	V	VI	VII
I		5.1	4.78	5.91	.5.94	5.26	4.57
II	5.69		4	4.76	6.1	4.97	5.41
III	5.38	4.47		4.63	5.03	4.6	4.47
IV	5.94	5	4.22		6	4.35	4.79
V	6.19	4.79	4.47	5.51		5.19	4.85
VI	5.04	5.44	4.72	5.07	5.56		4.5
VII	5.85	4.16	4.16	4.53	5.16	4.19	

(3) Theoretical Music Spaces (TMS)



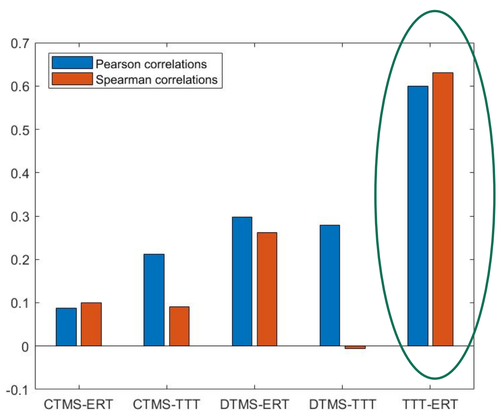
	I	II	III	IV	V	VI	VII
I		$\sqrt{2}$	$\sqrt{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
II	$\sqrt{2}$		2	1	$\sqrt{5}$	2	$\sqrt{8}$
III	$\sqrt{2}$	2		$\sqrt{5}$	1	$\sqrt{8}$	2
IV	1	1	$\sqrt{5}$		2	1	$\sqrt{5}$
V	1	$\sqrt{5}$	1	2		$\sqrt{5}$	1
VI	$\sqrt{2}$	2	$\sqrt{8}$	1	$\sqrt{5}$		2
VII	$\sqrt{2}$	$\sqrt{8}$	2	$\sqrt{5}$	1	2	

A (cognitive) representation of the tonal structure. Based on Euler original diatonic space.

Basic correlations

I calculated the basic correlations between the objects. A few representative results are shown.

CTMS – Chromatic Theoretical Musical Space.
DTMS – Diatonic Theoretical Musical Space.
ERT – Empirical Relatedness Table
TTT – Temperley Transition Table



The rule of normalizations

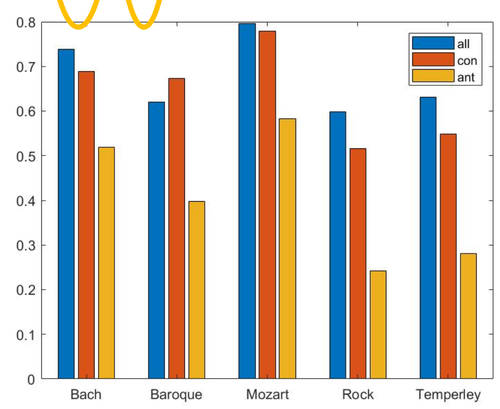
- **Overall normalization**: $\sum_i \sum_j T_{i,j} = 1$ (equivalent to raw data)
- **Raw normalization** (consequent): $\forall i, \sum_j T_{i,j} = 1$
- **Column normalization** (antecedent): $\forall j, \sum_i T_{i,j} = 1$

	I	II	III	IV	V	VI	VII
I		.143	.025	.309	.376	.12	.025
II	.274		.04	.032	.564	.04	.048
III	.063	.127		.34	.063	.383	.021
IV	.304	.132	.025		.38	.025	.132
V	.793	.021	.021	.039		.119	.005
VI	.155	.232	.069	.139	.286		.116
VII	.583	.001	.027	.138	.194	.055	

	I	II	III	IV	V	VI	VII
I		.079	.557	.879	.65	.647	.405
II	.218		.212	.022	.232	.051	.185
III	.014	.046		.063	.007	.136	.022
IV	.151	.108	.083		.01	.02	.318
V	.601	.026	.106	.025		.145	.018
VI	.014	.035	.042	.01	.013		.051
VII	.001	.000	.000	.000	.000	.000	

Comparing different normalizations in different corpora reveals that

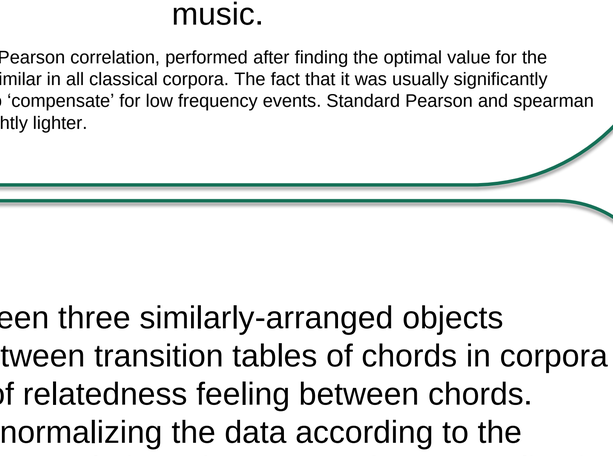
- The overall normalization is better than expected (and is usually the best normalization).
- Consequent normalization (rows) is better than antecedent normalization (columns) in agreement with previous research.

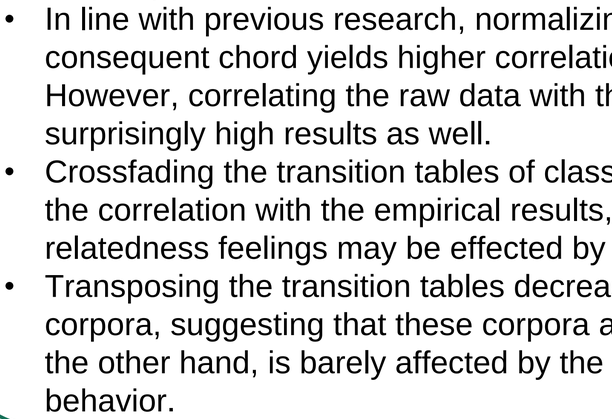


The tables with the overall normalization and the consequent normalization demonstrate stronger correlations than the antecedent normalized tables.
(Spearman correlations are shown, Pearson correlations behave roughly the same).

'Crossfading' Tables

- Using a "corrected Pearson" correlation reveals an improved correlation between the ERT and TT when crossfading the Rock TT with classical TTs.
- The average contribution of crossfading classical corpera was much lower (0.022, compared with 0.49).





- The correlations of most of the classical corpora showed a decline when crossfading the original matrices with their transposed table.
- The Baroque Corpus, however, displays an interesting behavior.
- The correlation of the Rock Corpus stays quite steady, inline with previous research, that found rock to be less 'directional' and 'asymmetrical' than classical music.

I used the term "Corrected Pearson" for an ordinal Pearson correlation, performed after finding the optimal value for the correlation corr(A.^c, B.^c). The parameter c was similar in all classical corpora. The fact that it was usually significantly smaller than 1, indicating that cognitive tendency to 'compensate' for low frequency events. Standard Pearson and spearman correlations showed similar tendency, although slightly lighter.

Conclusions

- Studying the correlations between three similarly-arranged objects uncovered high association between transition tables of chords in corpora and empirical measurements of relatedness feeling between chords.
- In line with previous research, normalizing the data according to the consequent chord yields higher correlations than antecedent normalization. However, correlating the raw data with the subject's feelings produces surprisingly high results as well.
- Crossfading the transition tables of classical and rock corpora can benefit the correlation with the empirical results, suggesting that participants' relatedness feelings may be effected by both corpora.
- Transposing the transition tables decreases the correlation in most classical corpora, suggesting that these corpora are 'directional'. The rock corpus, on the other hand, is barely affected by the process, implying a symmetrical behavior.

References:
Bharucha, J., & Krumhansl, C. L. (1983). The representation of harmonic structure in music: Hierarchies of stability as a function of context. *Cognition*, *13*(1), 63-102.
Lerdahl, F. (2004). *Tonal pitch space*. Oxford University Press. page 72.
Tymoczko, D. (2010). *A geometry of music: Harmony and counterpoint in the extended common practice*. Oxford University Press. page 231.
De Clercq, T., & Temperley, D. (2011). A corpus analysis of rock harmony. *Popular Music*, *30*(1), 47-70.
Jacoby, N., Tishby, N., & Tymoczko, D. (2015). An information theoretic approach to chord categorization and functional harmony. *Journal of New Music Research*, *44*(3), 219-244.

Acknowledgements: Ram Reuven, Andreas Hiada, Hadas Nahshon, Michal Goldstien.